Understanding Topological Order with PEPS

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Outlook

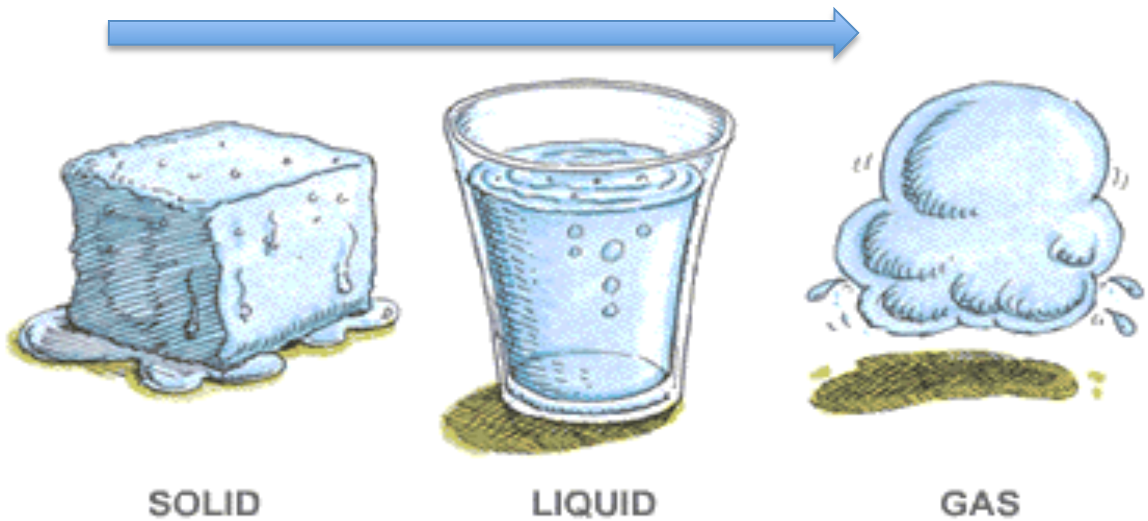
1. An introduction to topological order

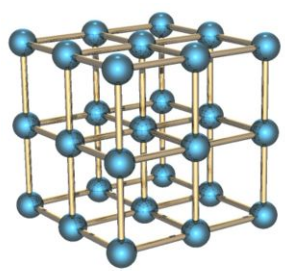
2. Topological order in PEPS

3. Open problems.

Phases. Order. Symmetries

Temperature





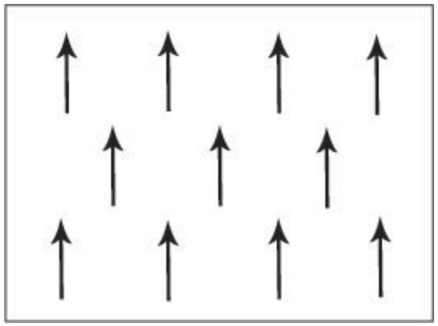
Order. Only lattice symmetry

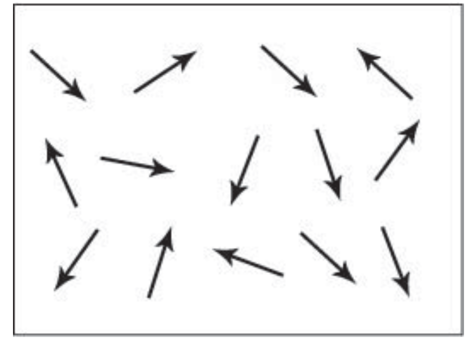
Disorder. Full translational symmetry

Phases. Order. Symmetries

Q-phase (T=0)?

Parameters in the Hamiltonian





Ferromagnetic state.
Some order.
Local SU(2) symmetry broken

Spin liquid. All symmetries

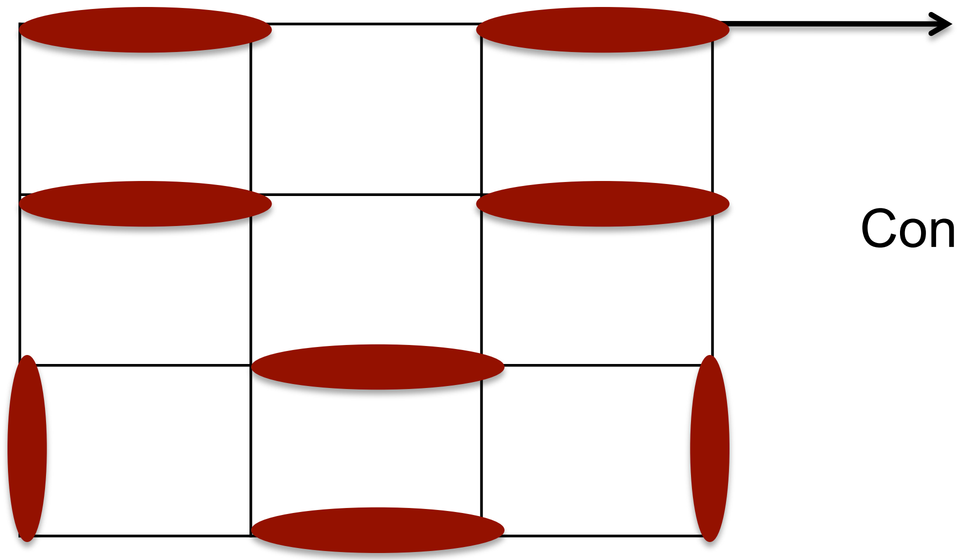
$$\langle Z_i \rangle = 0$$

$\langle Z_i \rangle = 1$ Magnetization per particle distinguishes the phases.
It is a local order parameter.

Landau Approach to Phases

There can be different types of order (ferromagnetic, antiferromagnetic, …).
They are characterized by a broken symmetry (detected by some order parameter).

80's. New type of order. Topological order. RVB. QDM



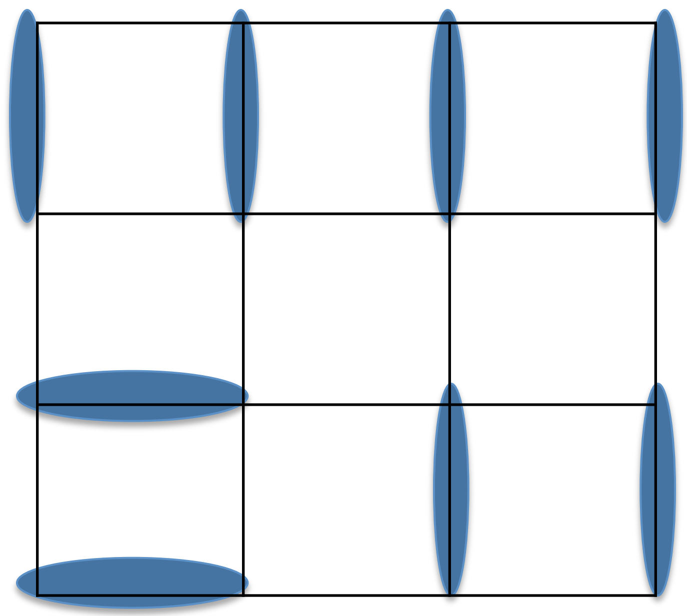
singlet $\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$

Configuration = covering of the lattice.

Configurations non-orthogonal.

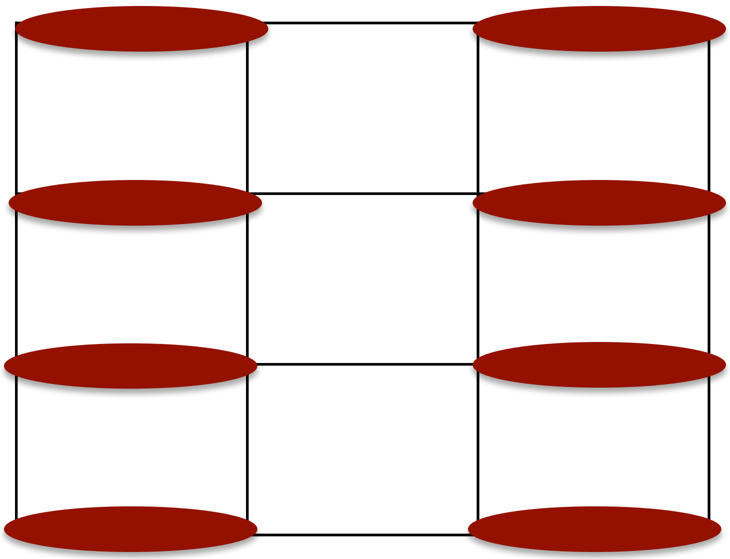
QDM = orthogonal "by definition" and we restrict to the Hilbert space spanned by the configurations.

Rokhsar-Kivelson 1988



Quantum Dimer Model

$$H_{QDM} = \sum -t\left(|\,\mathrm{I\,I}\,\rangle\langle\,=\,| + h.c.\right) + v\left(|\,\mathrm{I\,I}\,\rangle\langle\,\mathrm{I\,I}\,| + |\,=\,\rangle\langle\,=\,|\right)$$



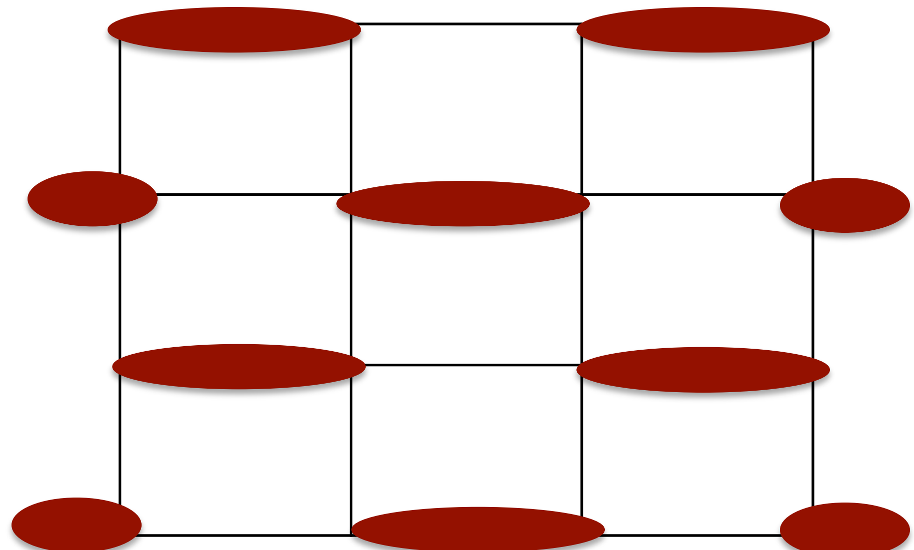
Column order

$$v/t \to -\infty$$

RK point

$$v/t = 1$$

$$|RVB\rangle \propto \sum_{config} |config\rangle$$



Staggered order

$$v/t \to \infty$$

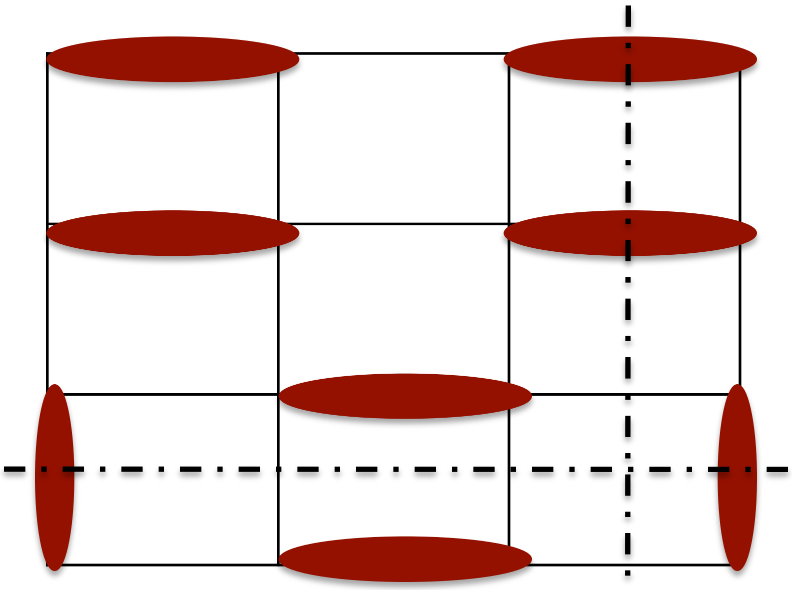
RVB state

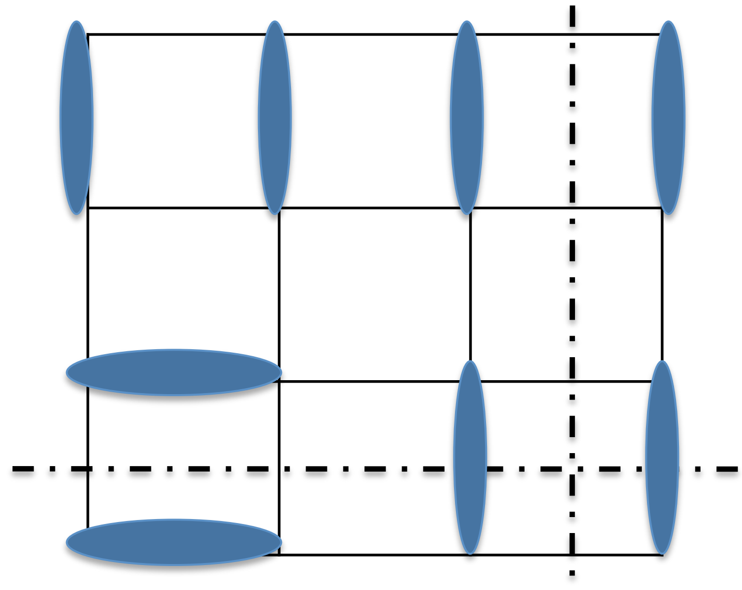
- The RVBS does not break any symmetry = spin liquid
- Postulated by Anderson (1987) to explain high Tc superconductivity.
- Candidate for a new (unobserved) phase: topological spin liquid

Real materials

Meng et al.
Nature 2010

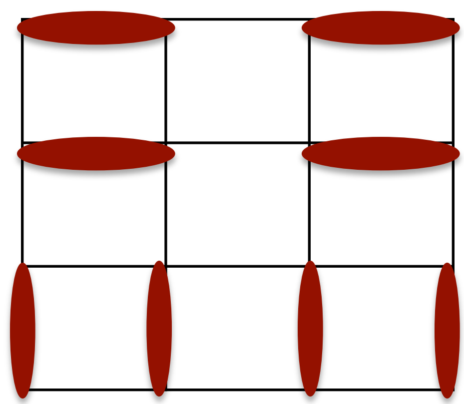
Topological order in the QDM

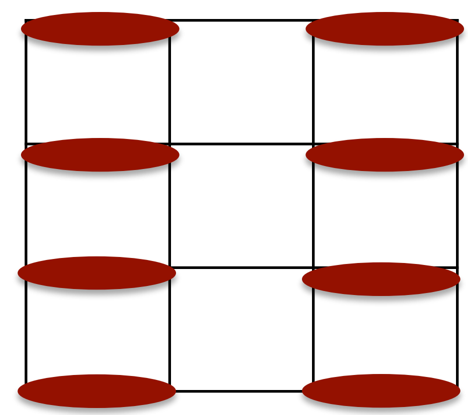




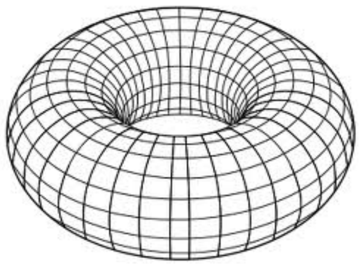
Number of cuts = even

One obtains all configurations from a reference one (column) by local resonating moves.





This can be changed if we change the topology. TORUS



Topological order in the QDM

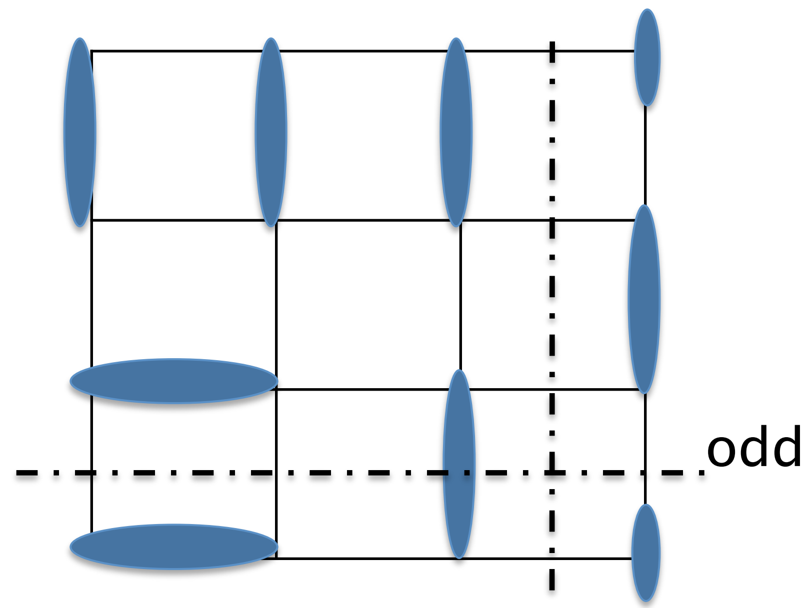
Different topological sectors.

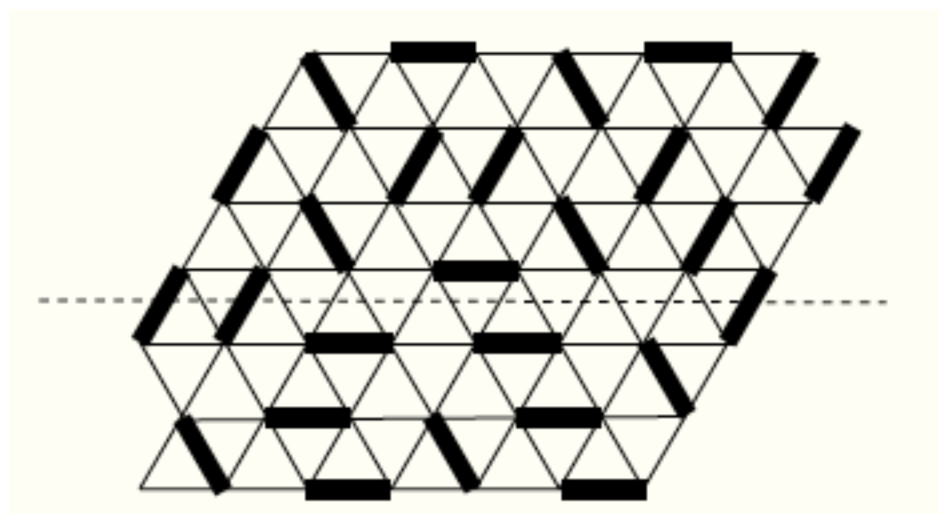
Within each one, all states related by local resonating moves.

No way to move between sectors with local resonating moves.

Sectors labeled by some winding numbers. In the triangular lattice = Parity of dimers intersecting the 2 reference lines (4 sectors).

At the RK point, GS = the RVB within each sector. Degeneracy = number of sectors

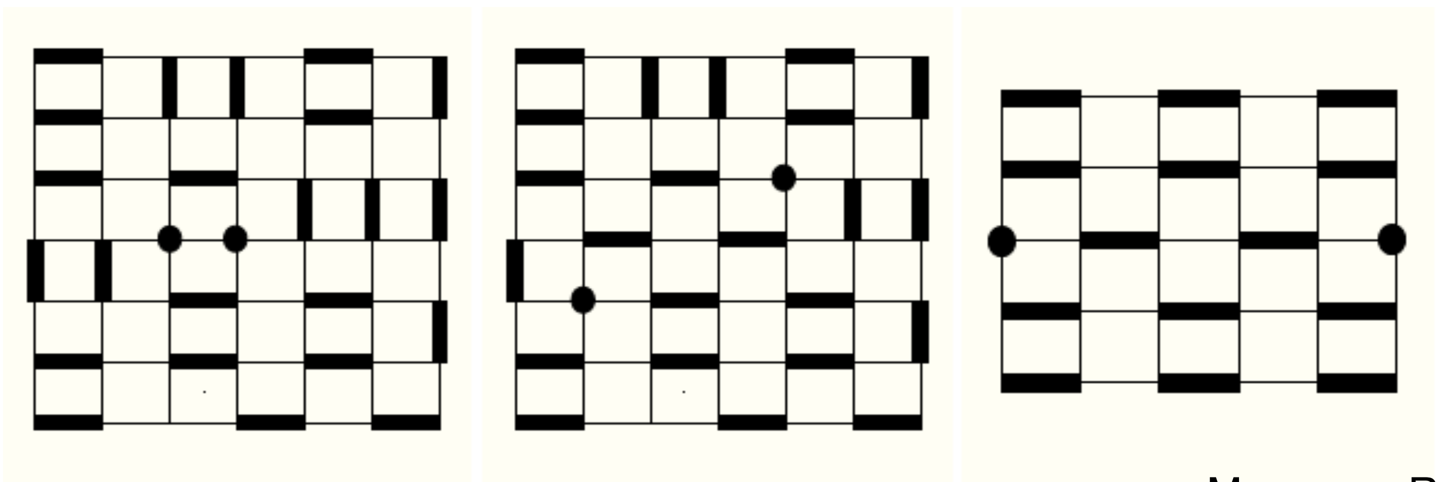




Moessner-Raman (2008)

Definition of topological order

1. Degeneracy of the Hamiltonian depends on topology
2. All GS are indistinguishable locally (no local order parameter).
3. To map between them you need a non-local operator.
4. Excitations behave like quasiparticles with anyonic statistics.



Moessner-Raman (2008)

5. There is an energy gap in the Hamiltonian.

Which properties do arise from 1-5?

Is there a systematic way to construct systems with 1-5?

Consequences of topological order

Topologically ordered systems are robust. Candidates for quantum memories. Information encoded in the topological sector.

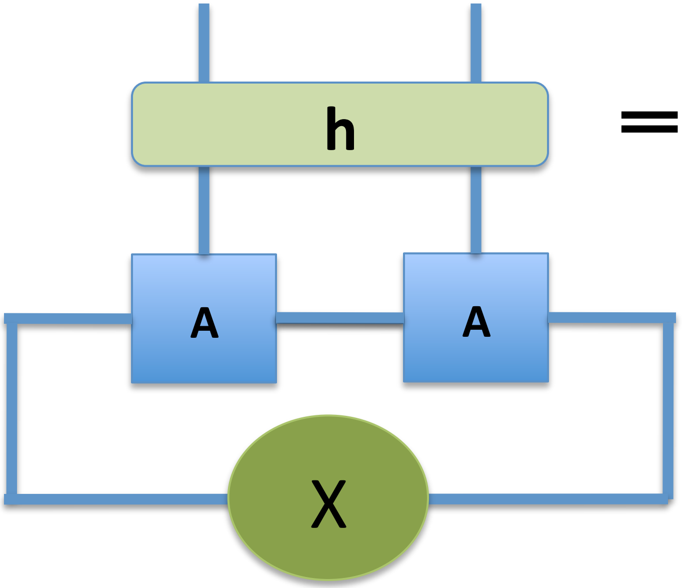
They are difficult to create.

Theorem (Bravyi-Hastings-Verstraete 2006): To create topological order with a (time-dependent) geometrically local Hamiltonian one needs time of the order of the size of the system.

Proof: Lieb-Robinson bounds.

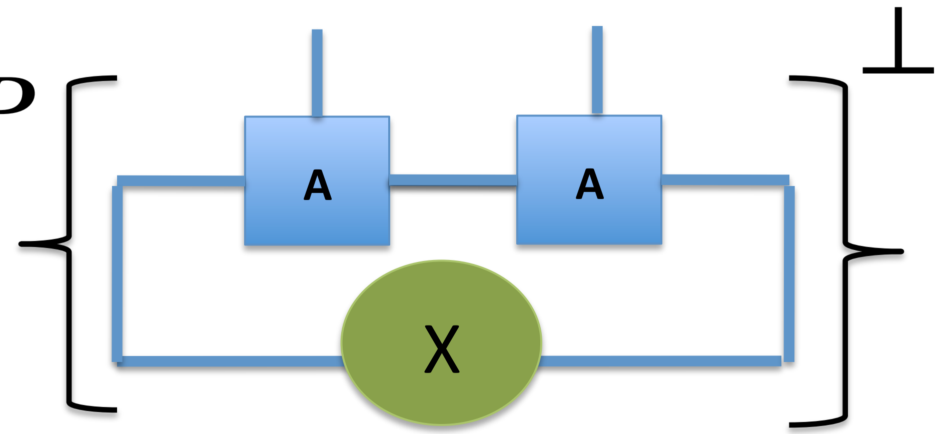
How to construct topologically ordered systems with PEPS

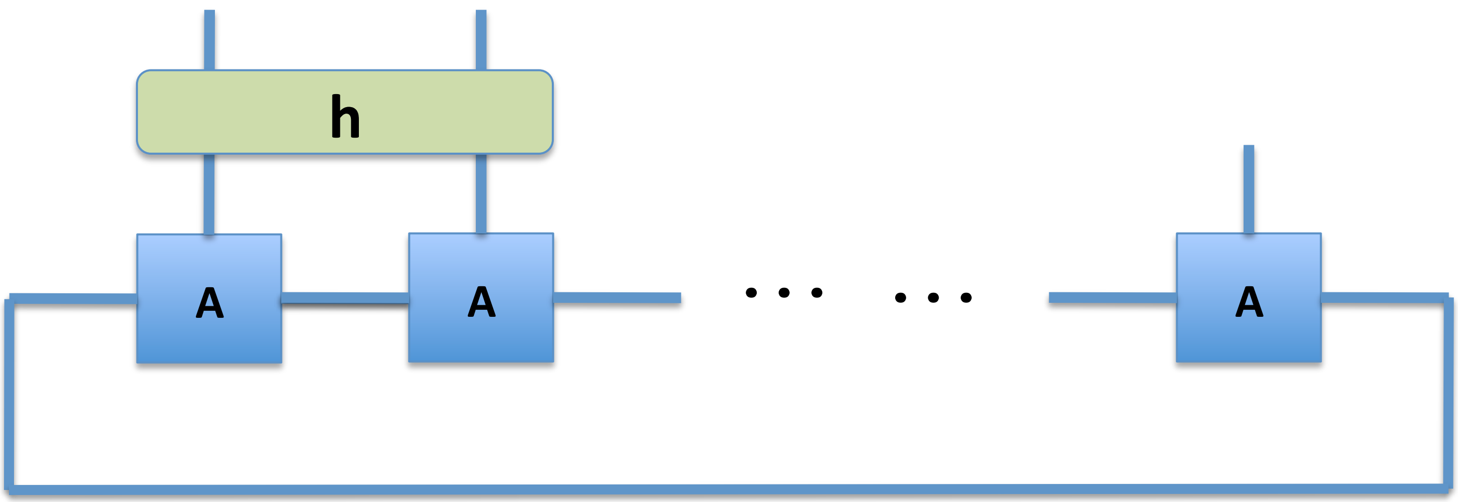
Reminder. Parent Hamiltonian in 1D



$= 0$

$h = P$





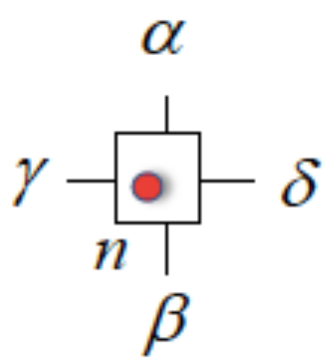
$= 0$

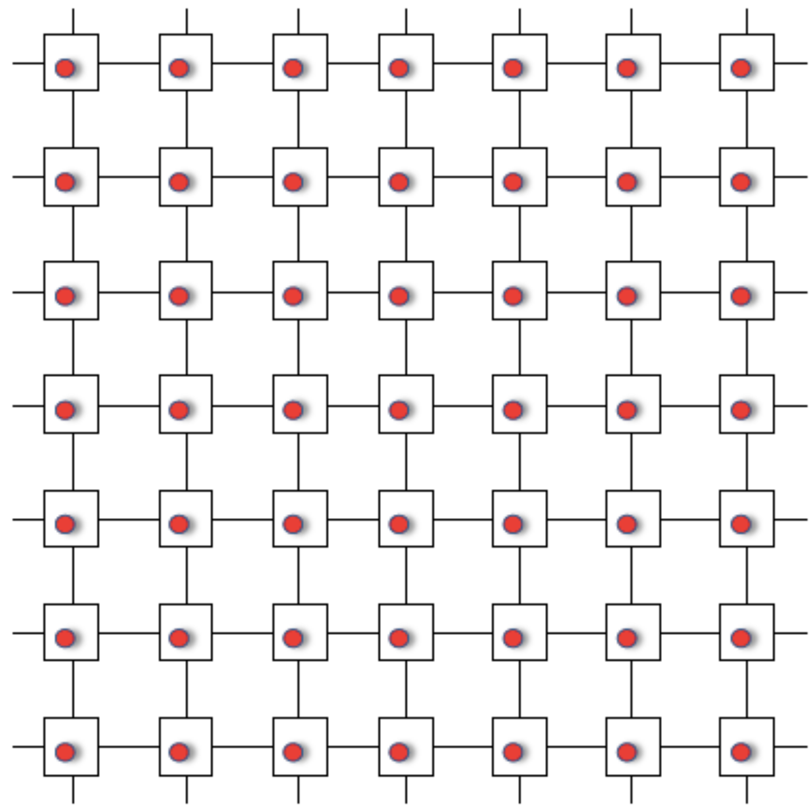
Parent Hamiltonian

$$H = \sum_i h_i \qquad H \geq 0 \qquad H|MPS\rangle = 0$$

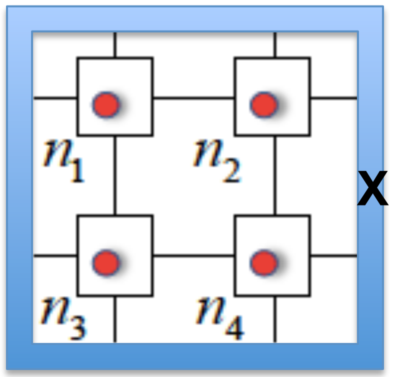
MPS is GS of H

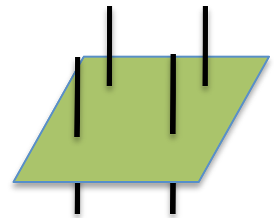
The same in 2D

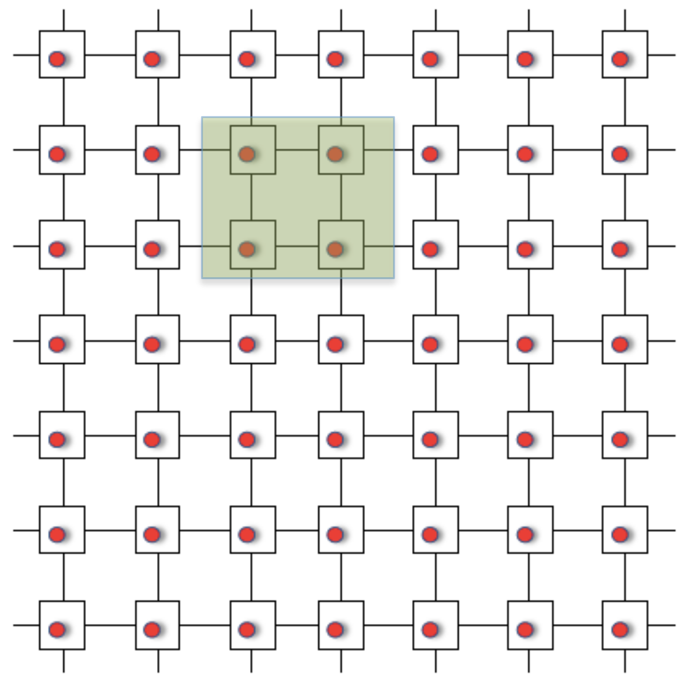
$A^n_{\alpha,\beta,\gamma,\delta} =$ 



Parent Hamiltonian

$h = P\left\{\right.$ 

$\left.\right\}^{\perp} =$ 



$= 0$

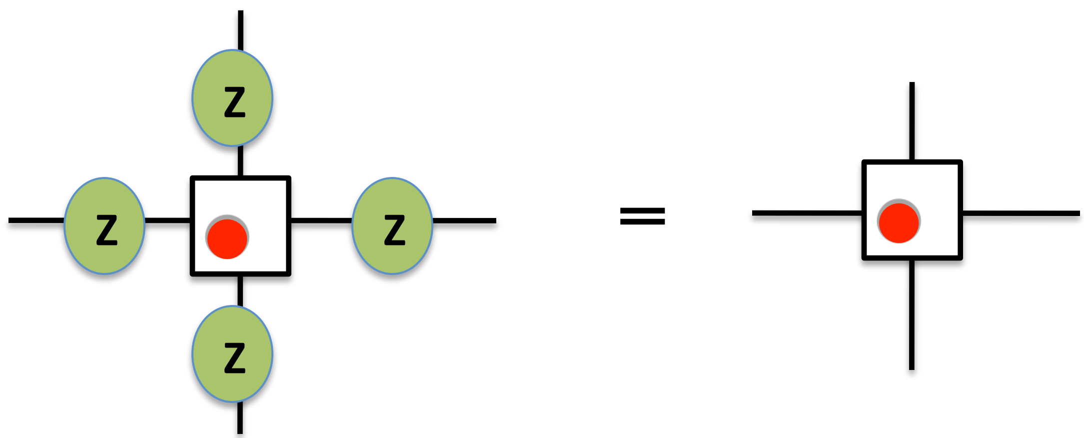
$$H = \sum_i h_i$$

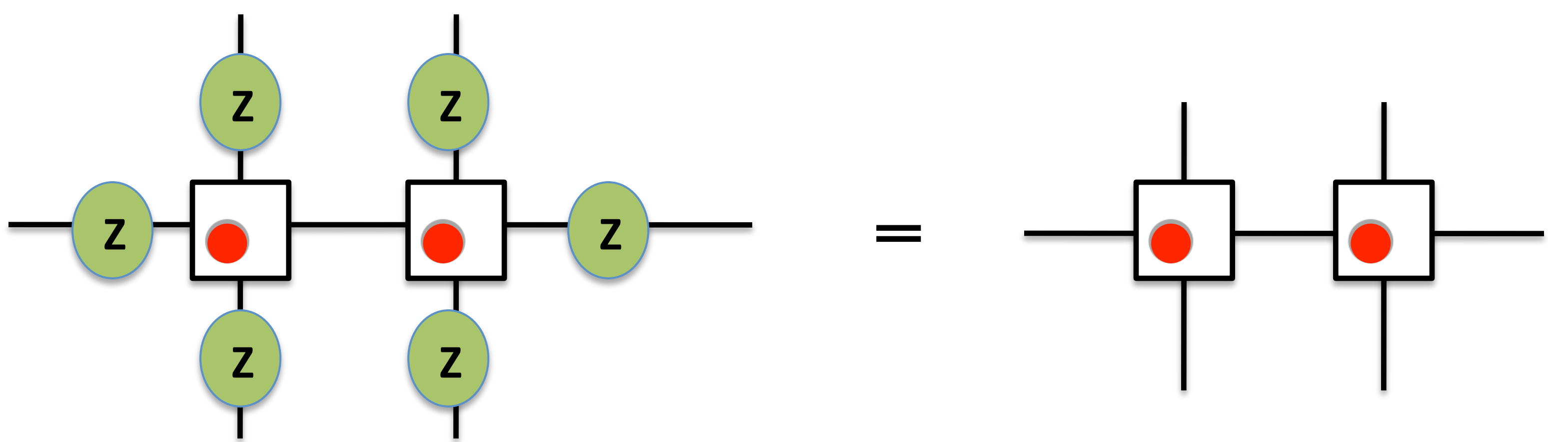
$$H \geq 0$$

$$H|PEPS\rangle = 0$$

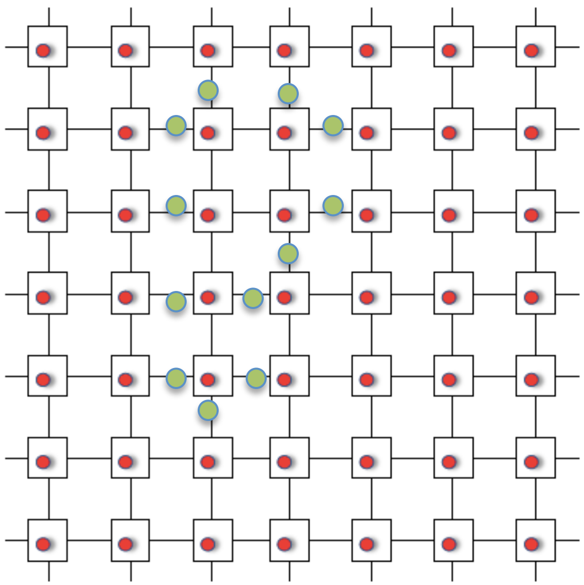
Topology in PEPS. Gauge symmetry

G any finite group. For example $G = Z_2 = \{1, Z\}$





Topology in PEPS. Gauge symmetry



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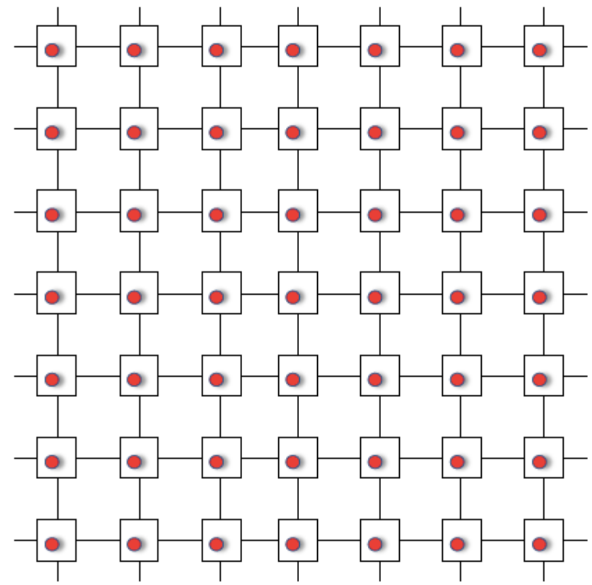
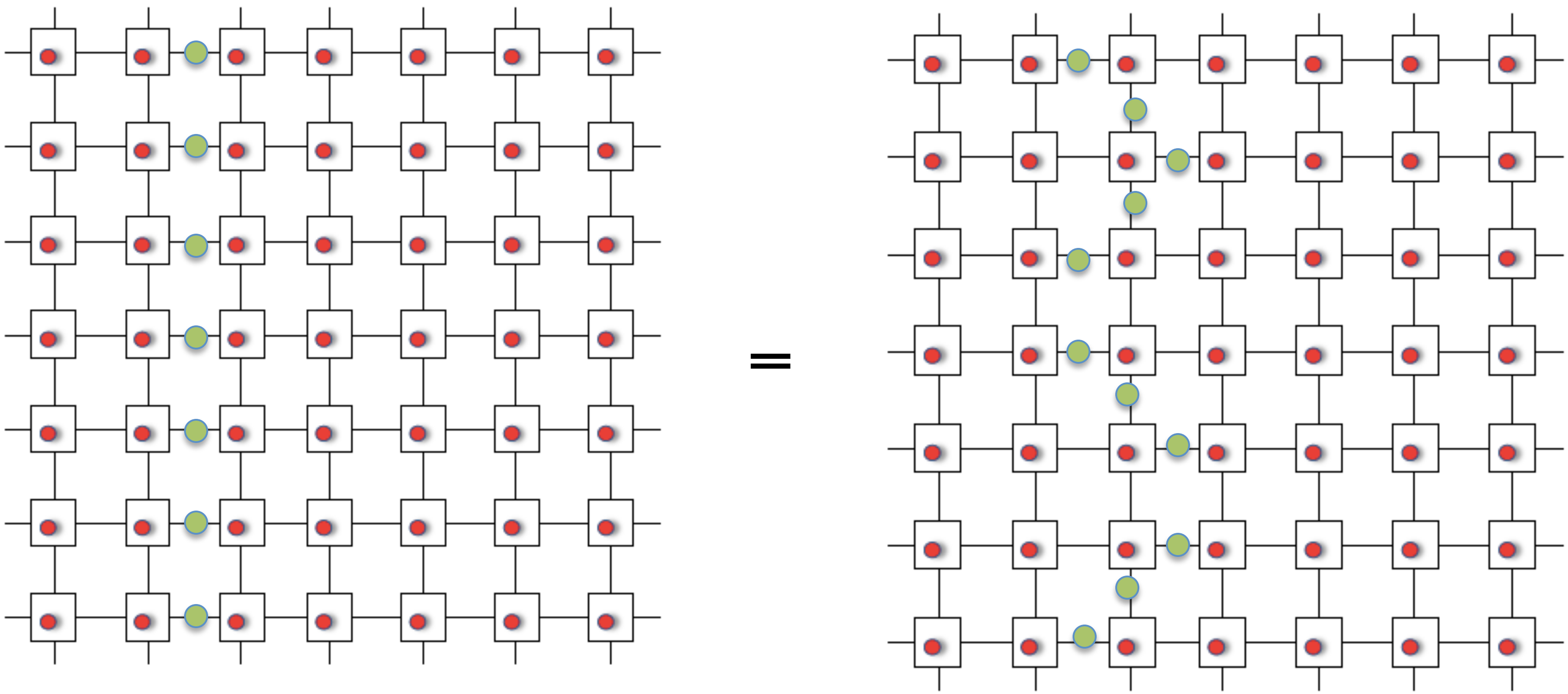


Contractible loops of Z vanish.

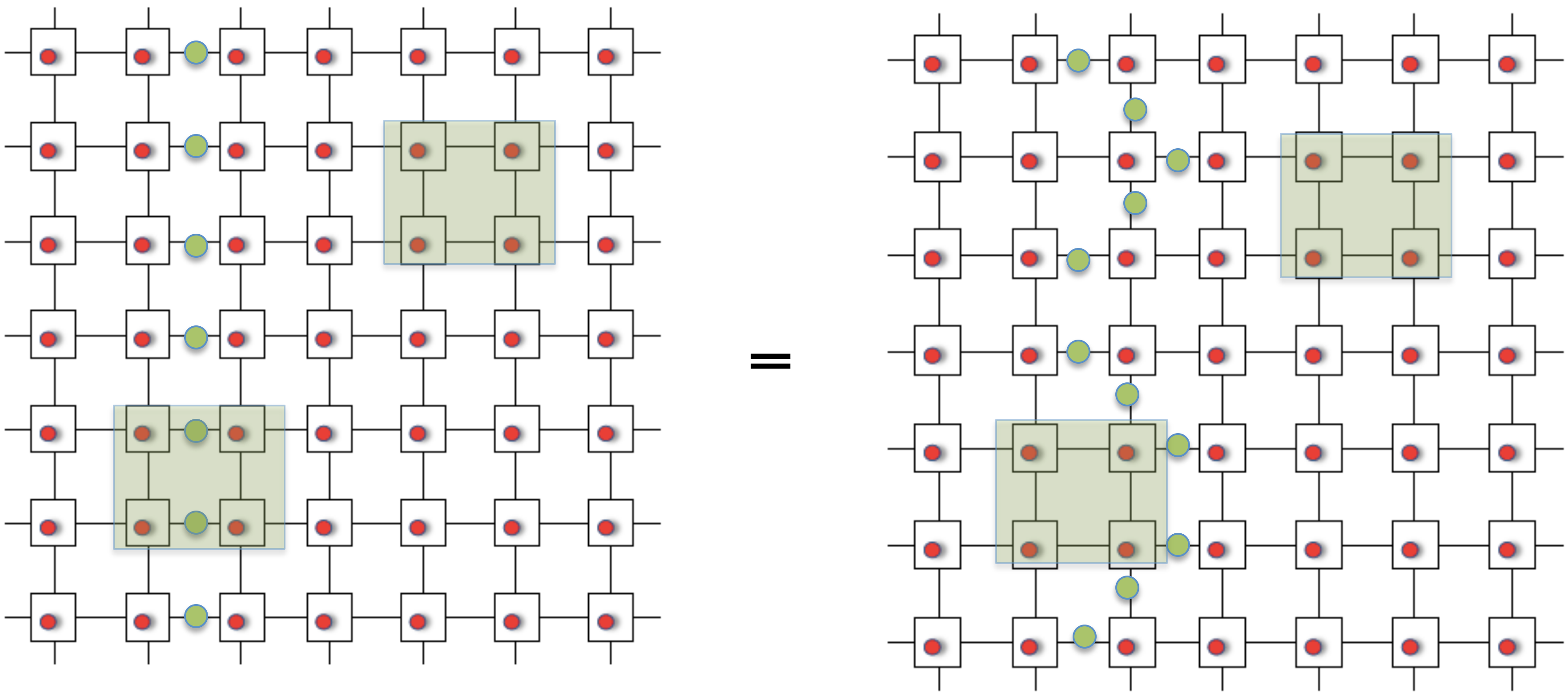
What about not contractible loops?

Topology in PEPS. Gauge symmetry



Non contractible loops can be arbitrarily deformed but they do not vanish.

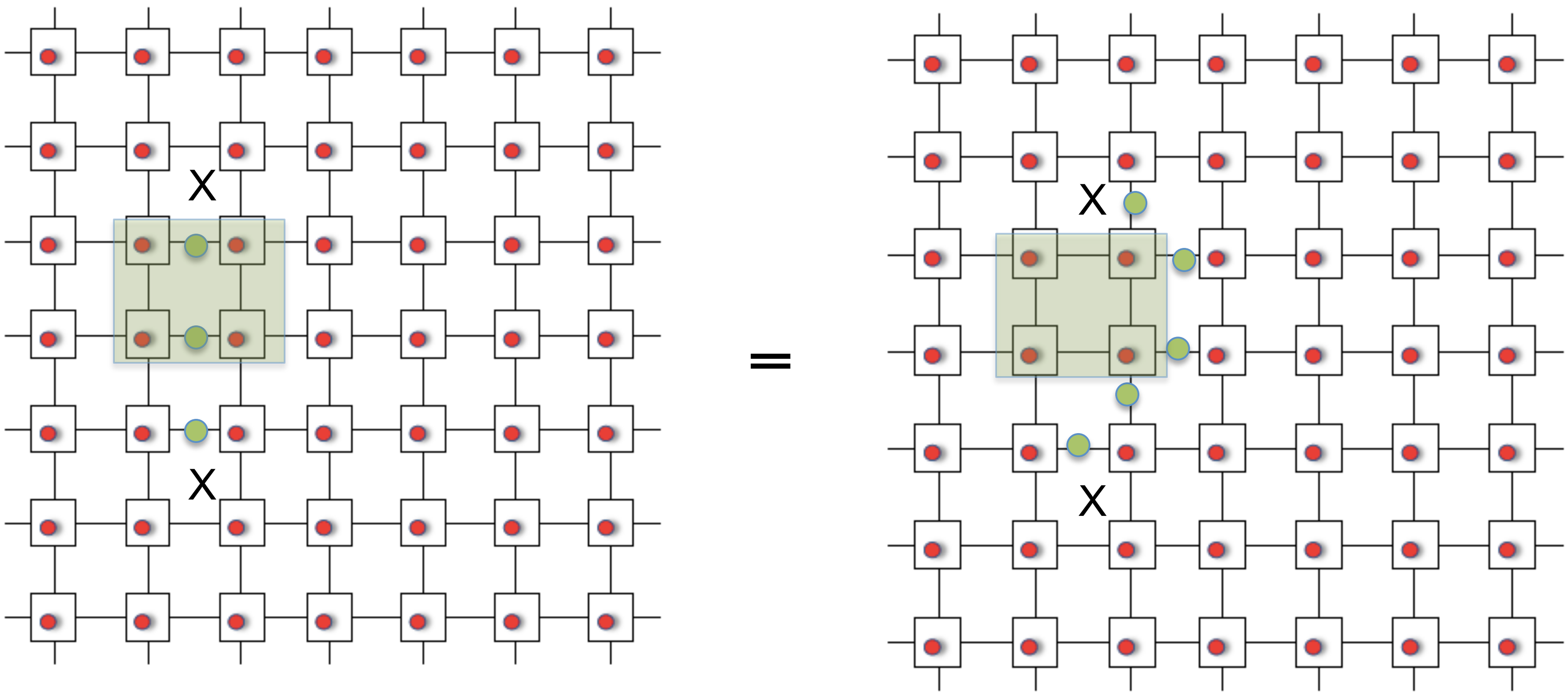
Topology in PEPS. Gauge symmetry



Non contractible loops can be arbitrarily deformed but they do not vanish.

New ground states of the parent Hamiltonian (which are locally equal).

Excitations = open strings



Open strings can be arbitrarily deformed except for the extreme points (quasi-particles).

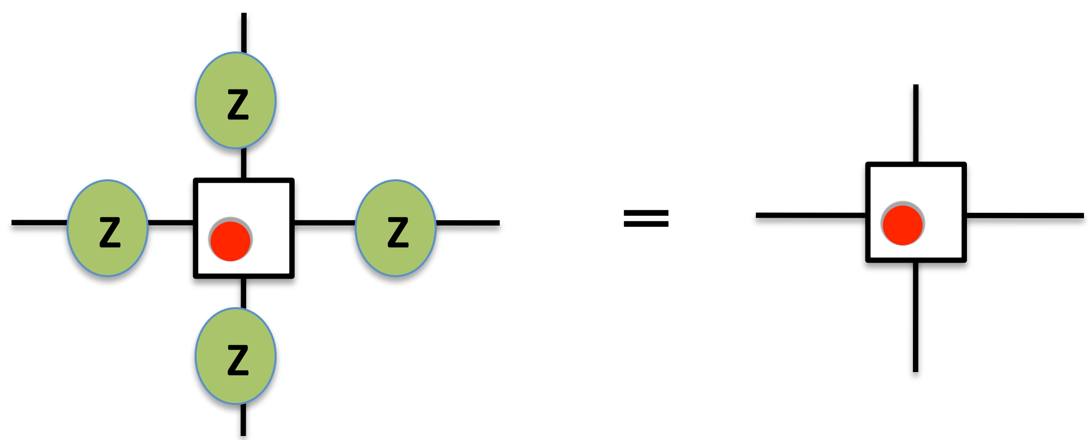
All of them have the same energy (=2). Quasi-particles can move freely.

We recover topological order

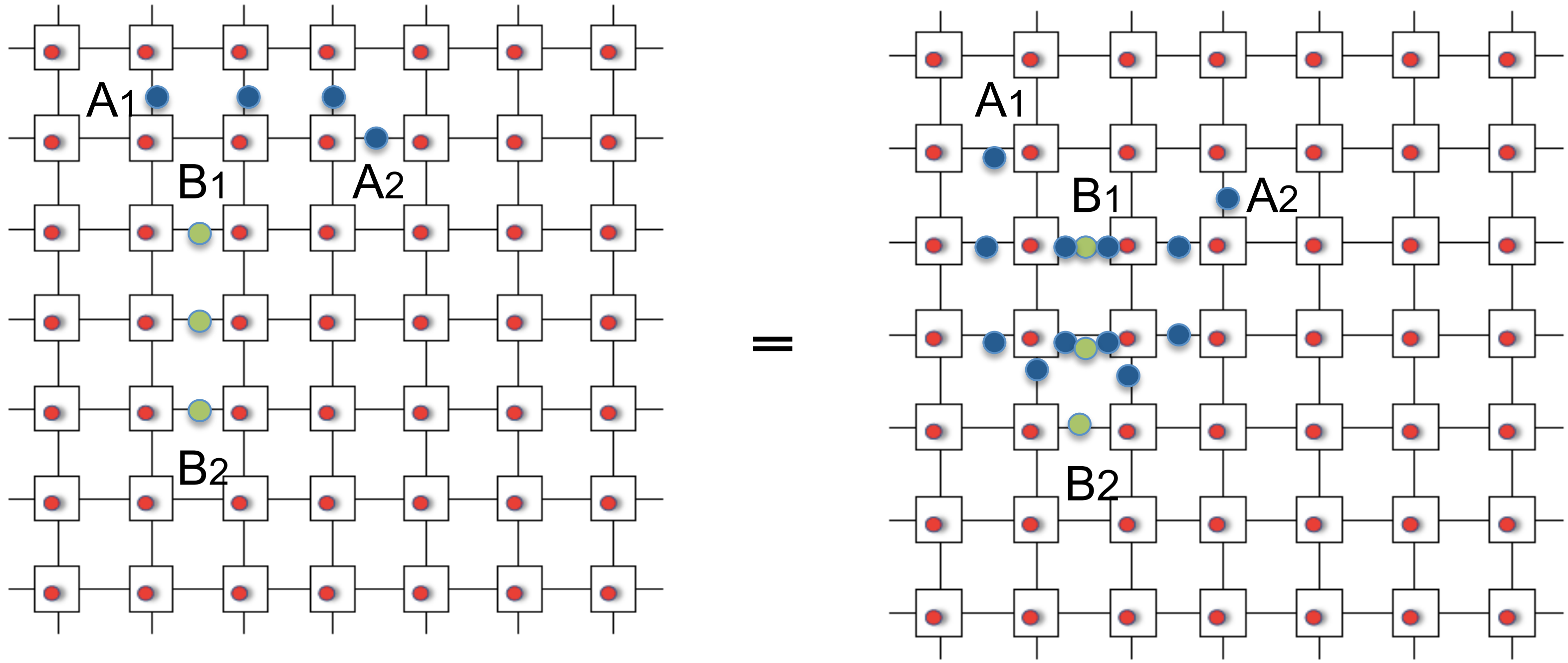
1. Degeneracy of the Hamiltonian depends on topology
2. All GS are indistinguishable locally (no local order parameter).
3. Excitations behave like quasiparticles with anyonic statistics.
4. To move between GS: non-local operator.

Indeed one does need some extra condition for this to hold (*G-isometric*)

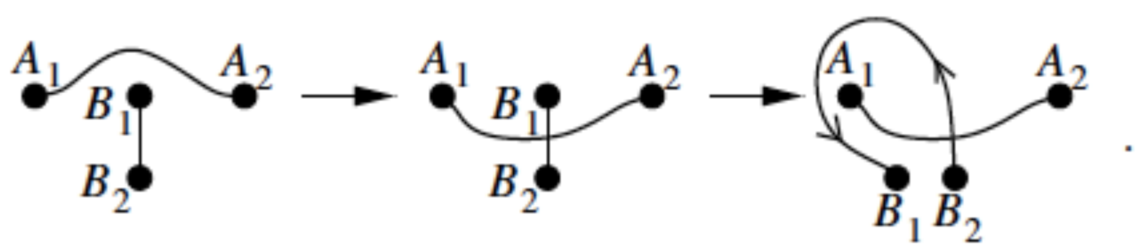
on top of



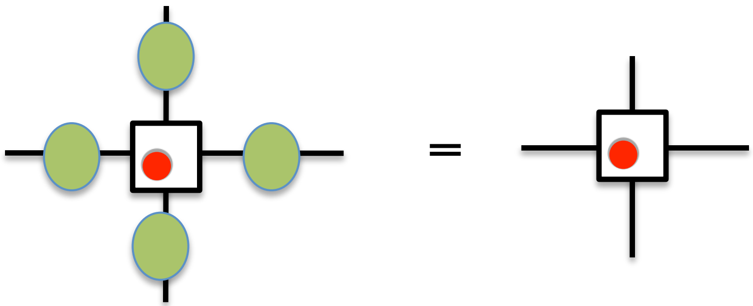
Anyonic statistics (G non-abelian)



Moving one excitation around another one has a non-trivial effect.



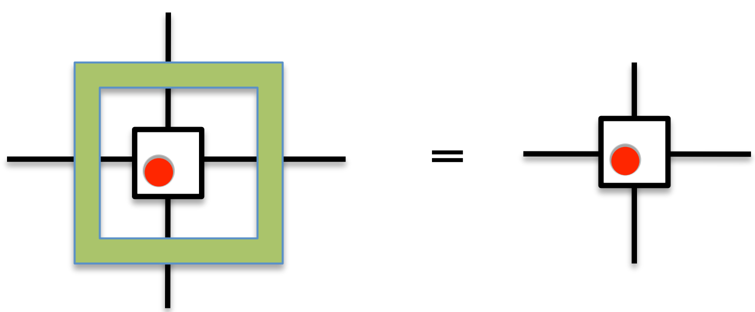
More and more weird models

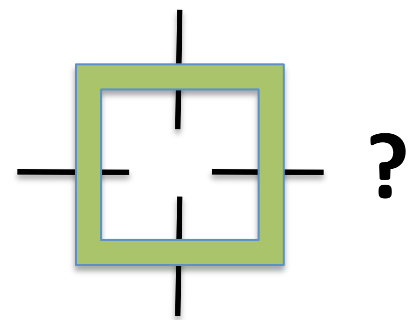


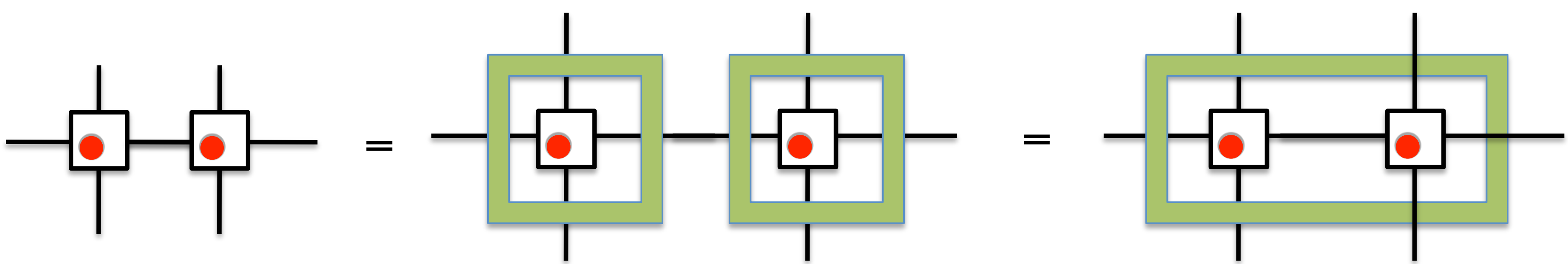
$G = Z_2$ Toric code

$G = S_3$ Universal topological quantum computation

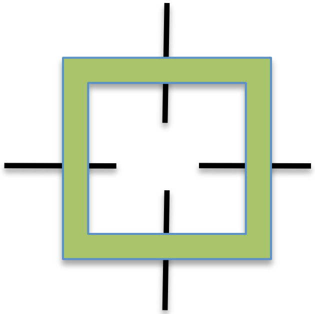
Beyond groups



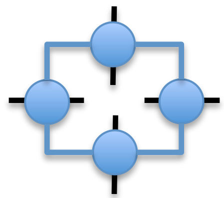




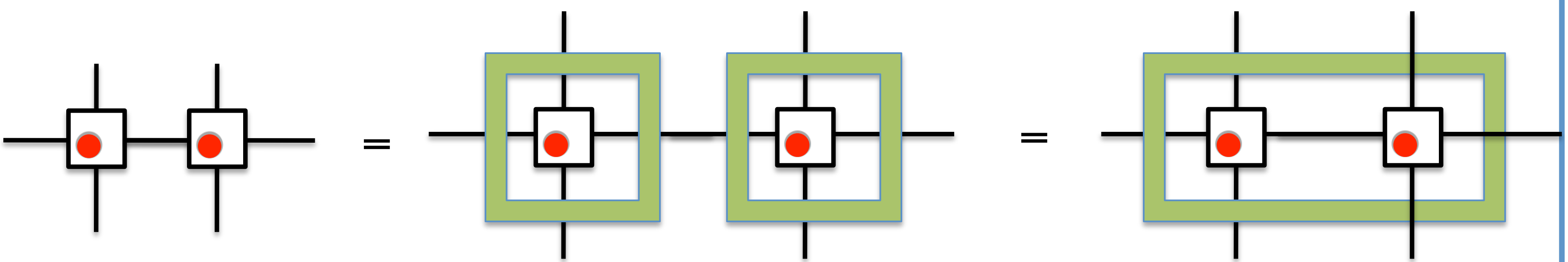
Weird models. All models?



$$= \pi^{\otimes 4}(S \otimes 1 \otimes S \otimes 1)\Delta^{3}(h)$$ Buerschaper et al.

=  String-net models. Sahinoglu et al.

Can one classify all possibilities? Is



the only possibility to get topological order in PEPS?

Is there a PEPS in any phase?

What happens in the 3D case?