Advanced Quantum Information Theory

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Theorems and Definitions

Chapter 0

Notation and Terminology

0.1 Miscellaneous

s.t.	"such that"
w.l.o.g.	"without loss of generality"

0.2 Hilbert spaces

$\bar{x}$	complex conjugate of x
$\mathbb{C}^2$	2-dimensional Hilbert space ("qubit")
$\mathbb{C}^d$	d-dimensional Hilbert space ("qudit")
$\mathcal{H}^{\otimes n}$	$\bigotimes_1^n \mathcal{H} = \underbrace{\mathcal{H} \otimes \mathcal{H} \otimes \ldots \otimes \mathcal{H}}_{n \text{ copies}}$ (e.g. n qubits: $(\mathbb{C}^2)^{\otimes n}$)
$\text{span}\{\lvert\psi_i\rangle\}$	linear subspace spanned by $\lvert\psi_1\rangle, \lvert\psi_2\rangle, \ldots$

0.3 Computer science

$f(n) = O(g(n))$	$\exists c > 0, N > 0$ s.t. $\forall n > N : \lvert f(n)\rvert < c\lvert g(n)\rvert$
$f(n) = \Omega(g(n))$	$\exists c > 0, N > 0$ s.t. $\forall n > N : \lvert f(n)\rvert > c\lvert g(n)\rvert$
$\{0,1\}^n$	set of all strings of 0's and 1's of length n
$\{0,1\}^*$	set of all strings of 0's and 1's of arbitrary finite length

0.4 Operators

$\mathcal{B}(\mathcal{H})$	bounded operators on Hilbert space $\mathcal{H}$ (recall: X bounded $\Leftrightarrow \forall \lvert\psi\rangle : \lVert X \lvert\psi\rangle\rVert < \infty$)
$\mathbb{1}$	identity operator
$\Pi^{(0)}$	projector $\Pi^{(0)} = \lvert 0\rangle\langle 0\rvert$
$\Pi^{(1)}$	projector $\Pi^{(1)} = \lvert 1\rangle\langle 1\rvert$
$\ker X$	kernel $\ker X := \mathrm{span}\{\lvert\psi\rangle : X \lvert\psi\rangle = 0\}$
$\mathrm{supp}\, X$	support* $\mathrm{supp}\, X := (\lvert X\rangle)^{\perp}$
$X^\dagger$	Hermitian conjugate (or adjoint)
$H^\dagger = H$	Hermitian (or self-adjoint) operator
$H > 0$	positive operator (implies Hermitian): $H > 0 \Leftrightarrow \forall \lvert\psi\rangle : \langle\psi\rvert H \lvert\psi\rangle > 0$
$H \geq 0$	positive-semidefinite (or non-negative) operator: replace "$>$" by "$\geq$" in definition of positive operator
$A \geq B$	$A - B \geq 0$ (A, B Hermitian operators)
$A \geq c$	$A - c\mathbb{1} \geq 0$ (A Hermitian operator, $c \in \mathbb{R}$)

0.5 Hamiltonians

*Warning: quantum information terminology! Sometimes called "co-image" elsewhere.