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Advanced Quantum Information Theory

Introduction & Course Outline

- Will only need very basic quant-inf. knowledge
- Lecture notes; examples sheets

Outline:

QIT course, "preliminaries" link, Nielsen + Chuang

0. Background (Complexity Theory)
   Kitaev, Shen, Vyalyi – everything needed here
   Nielsen + Chuang – almost everything (no QMA)
   Arora + Barak – to go further (classical)

I. Hamiltonian Complexity
   Closely follow Kitaev book

II. Many-body quantum systems
   Research papers, Hastings' Les Houches notes

Notation and Terminology

Basic:

- $\mathbb{C}^2$ = Hilbert space of dim 2 ("qubit")
- $\mathbb{C}^d$ = d-dim Hilbert space ("qudit")
- $\mathcal{H}^{\otimes n} \equiv \bigotimes_{i=1}^{n} \mathcal{H}$ e.g. n-qubits: $(\mathbb{C}^2)^{\otimes n}$
- $\text{span}\{|\psi_i\rangle\}$ = linear subspace spanned by $|\psi_1\rangle, |\psi_2\rangle, \ldots$
- Big-O notation:

$$f(x) = \mathcal{O}(g(x))$$
$$\Leftrightarrow \exists\, c, n > 0 \text{ s.t. } \forall x > n: \; f(x) < c\, g(x)$$
$$f(x) = \Omega(g(x))$$
$$\Leftrightarrow \exists\, c, n > 0 \text{ s.t. } \forall x > n: \; f(x) > c\, g(x)$$

Operators:

- $\mathcal{B}(\mathcal{H})$ = bounded operators on Hilbert space $\mathcal{H}$

  (Recall: X bounded $\Leftrightarrow$ $\| X|\psi\rangle \| < \infty$.
  All linear operators on finite-dim $\mathcal{H}$ bounded)

- $\mathbb{1}$ = identity operator

- $\Pi^{(0)} = |0\rangle\langle 0|$, $\Pi^{(1)} = |1\rangle\langle 1|$ (orthog. projectors)

- $\ker A := \operatorname{span}\{ |\psi\rangle : A|\psi\rangle = 0 \}$ (kernel)

- $\operatorname{supp} A := (\ker A)^{\perp}$ ("support")

  Warning: QIT terminoloy! "co-image" elsewhere

- positive operator $H > 0$ ($\Rightarrow H^{\dagger} = H$)

  $\Leftrightarrow$ $\forall |\psi\rangle$ $\langle\psi| H |\psi\rangle > 0$

- positive semi-definite: replace '$>$' with '$\geq$'

- "$A \geq B$" $\Rightarrow$ $A - B \geq 0$, "$A \geq c$" $\Rightarrow$ $A - cI \geq 0$.